Estimation of upcrossing rate of mean wind speeds from joint probability distribution of Weibull variables

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SUMMARY:
The upcrossing rate method is one of the most popular approaches to estimating the extreme mean wind speed based on the mean wind speed time history. Considering the mean wind speed time history as the discrete random process, the upcrossing rate, which takes the unit time equal to the sampling time interval, can be expressed in terms of the joint probability distribution of this random process and this random process with a lag equal to the sampling time interval. In this study, the mean wind speed is modelled by the Weibull distribution, and the Gaussian Copula is introduced to obtain the model of the joint probability distribution of Weibull variables. The upcrossing rate of the mean wind speed at a specific level can then be estimated. The proposed method is compared with the simulated data and the analytical formula of the upcrossing rate of the discrete standard Rayleigh process. The comparing results suggest the proposed method has a high accuracy for estimating the upcrossing rate.

Keywords: upcrossing rate, mean wind speed, joint probability distribution

1. INTRODUCTION

The estimation of the extreme mean wind speed has a pivotal role in the structural wind-resistance design. Estimating the extreme mean wind speed from short-term mean wind speed records, such as about a few years of records, is necessary when yearly maximum mean wind speed samples are insufficient. The upcrossing rate method is one of the most popular approaches to estimating the extreme mean wind speed from mean wind speed time history. Considering the mean wind speed process as a discrete random process, the upcrossing rate can be developed from the joint probability distribution function (PDF) of this process and this process with a lag equal to the sampling time interval. Based on this definition, the analytical formula of the upcrossing rate of the discrete standard Rayleigh process was established from the joint PDF of standard Rayleigh variables; and the joint PDF is obtained by decomposing the standard Rayleigh variable into two orthogonal standard Gaussian variables (Harris, 2017).

Given the traditionally Weibull-modelled mean wind speed (Harris and Cook, 2014), this study introduces the Gaussian Copula (Nelsen, 2006) to obtain the model of the joint PDF of Weibull variables and then estimates the upcrossing rate from this joint PDF. The upcrossing rate estimated by the proposed method is compared with the analytical formula (Harris, 2017) and the simulated data for investigation and validation.

2. UPCROSSING RATE OF MEAN WIND SPEED MODELLED BY WEIBULL DISTRIBUTION

For a random process $X(t)$, the upcrossing rate of it at a specific level x can be defined as the expectation of the occurrence number of the event that $X(t)$ crosses up x in the unit time. Consider $X(t)$ is discrete with the sampling time interval of $\Delta\tau$, and the occurrence of this event in $\Delta\tau$ can be expressed as $X(t) \le x$ and $X(t+\Delta\tau) > x$. Then, the upcrossing rate of $X(t)$ at x is equal to the occurrence number of this event divided by the total number of $\Delta\tau$, and that is just the cumulative probability of $X(t) \le x$ and $X(t+\Delta\tau) > x$. Thus, the upcrossing rate of $X(t)$ at x with unit time of sampling interval of $\Delta\tau$, $\vartheta(x)$, can be expressed as

$$\vartheta(x) = \int_{-\infty}^{x}\int_{x}^{+\infty} f_{X_1X_2}(x_1,x_2)\,dx_2dx_1 \quad (1)$$

where X_1 and X_2 are the random variables corresponding to $X(t)$ and $X(t+\Delta\tau)$, respectively; $f_{X_1X_2}(x_1,x_2)$ is the joint probability distribution function (PDF) of X_1 and X_2 with the correlation coefficient $\rho_{X_1X_2}$, and the $\rho_{X_1X_2}$ is equal to the normalized autocorrelation function with a lag of $\Delta\tau$, for process $X(t)$.

Consider the mean wind speed process $X(t)$ has the Weibull distribution with the cumulative distribution function (CDF) of $F_X(x) = 1 - \exp[-(x/c)^k]$ and PDF of $f_X(x) = f_X(x) = kx^{k-1}\exp[-(x/c)^k]/c^k$, where c and k are respectively the scale and shape parameters. To get the joint PDF of Weibull variables, the Gaussian Copula is introduced to obtain the joint CDF of Weibull variables X_1 and X_2 with the correlation coefficient $\rho_{X_1X_2}$ as

$$F_{X_1,X_2}(x_1,x_2) = \Phi_{Z_1,Z_2}(z_1,z_2) \quad (2)$$

where $\Phi_{Z_1,Z_2}(z_1,z_2,)$ is the joint CDF of standard Gaussian variables Z_1 and Z_2 with correlation coefficient $\rho_{Z_1Z_2}$; $z_1 = \Phi_{Z_1}{}^{-1}[F_{X_1}(x_1)]$, $z_2 = \Phi_{Z_2}{}^{-1}[F_{X_2}(x_2)]$, and $\Phi_{Z_i}{}^{-1}[\cdot]$ $(i = 1,2)$ is the inverse of the CDF of the standard Gaussian variable. The coefficient $\rho_{Z_1Z_2}$ can be obtained by the relationship between $\rho_{X_1X_2}$ and itself as (Liu and Kiureghian, 1986)

$$\rho_{X_1X_2} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x_1-\mu_1)(x_2-\mu_2)/(\sigma_1\sigma_2)\cdot\varphi_{Z_1,Z_2}(z_1,z_2,\rho_{Z_1Z_2})dz_1dz_2 \quad (3)$$

where $\varphi_{Z_1,Z_2}(z_1,z_2)$ is the joint PDF of variables Z_1 and Z_2; $\mu_1 = \mu_2$ and $\sigma_1 = \sigma_2$ are the mean and stand deviation of the $X(t)$, respectively. The joint PDF $f_{X_1X_2}(x_1,x_2)$ in Eq.(1) is the derivative function of $F_{X_1,X_2}(x_1,x_2)$ and can be obtained from Eq.(2) as

$$f_{X_1X_2}(x_1,x_2) = \partial^2F_{X_1,X_2}(x_1,x_2)/(\partial x_1\,\partial x_2) = f_{X_1}(x_1)f_{X_2}(x_2)\,\varphi_{Z_1,Z_2}(z_1,z_2)/[\varphi_{Z_1}(z_1)\varphi_{Z_2}(z_2)] \quad (4)$$

where φ_{Z_i} is the PDF of the standard Gaussian variable.

3. COMPARISON AND VALIDATION

The analytical formula of the upcrossing rate of the discrete standard Rayleigh process, which depends on the normalized autocorrelation function of the process, was derived from Eq.(1) (Harris, 2017). The upcrossing rate of the Weibull process, which depends on the normalized autocorrelation function and model parameters of the distribution of the process, was also developed based on the translation process method (This paper is under review). These two methods are compared with the method proposed in this study.

3.1. Comparison with Analytical Formula

The upcrossing rates estimated by the proposed method and the analytical formula are compared in this section. The scale and shape parameters of the Weibull distribution are set as $\sqrt{2}$ and 2 to reduce the Weibull distribution into the standard Rayleigh distribution, for the comparison. There is an obvious difference between results of the proposed method and the analytical formula at a low level and at a high level with small correlation coefficient ρ_{R_1,R_2}, as shown in Fig.1.

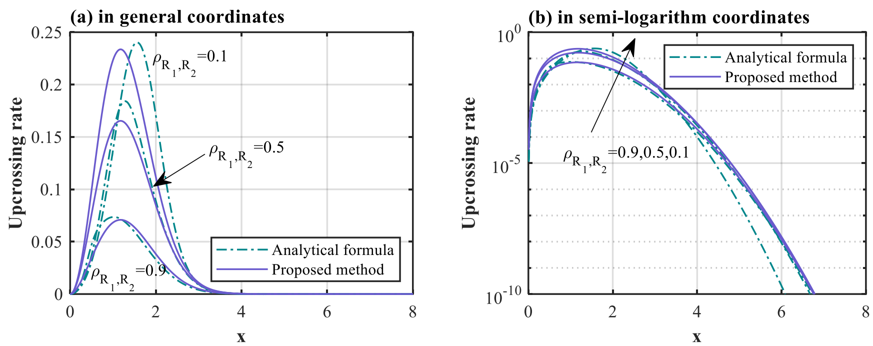

Figure 1. The comparison of upcrossing rates of the analytical formula and the proposed method for the standard Rayleigh process with different correlation coefficients $\rho_{R_1R_2}$

The uncorrelated white noise following the standard Rayleigh distribution is simulated to validate the effectiveness of the proposed method as shown in Fig.2. The proposed method performs well at both low level and high level. It should be noted that the difference between the upcrossing rates of the proposed method and the analytical formula will decrease with the correlation coefficient ρ_{R_1,R_2}.

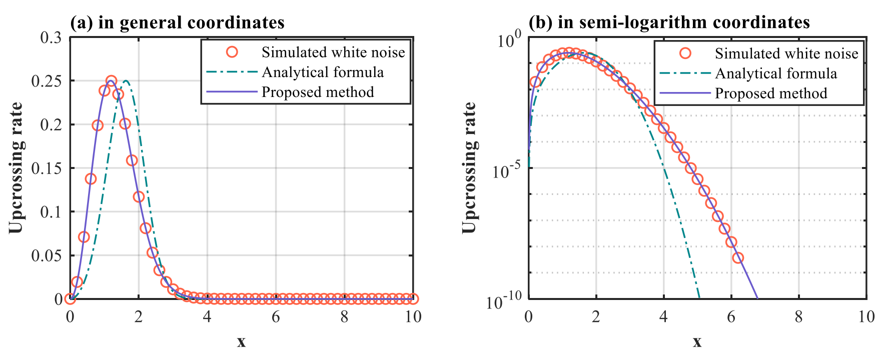

Figure 2. The comparison of the upcrossing rates of the simulated white noise following the standard Rayleigh distribution, the analytical formula and the proposed method

3.2. Validation by Simulated Mean Wind Speeds with Weibull Distribution

A total of 20000 years of 10-min mean wind speed timeseries is numerically generated by the spectral representation method (SRM) (Shinozuka and Deodatis, 1991; Liu et al., 2020) from wind speed spectrum (Van der Hoven, 1957) and the target Weibull distribution with scale parameter $c = 2.35$ and the shape parameter $k = 1.50$, to validate the effectiveness of the proposed method. The proposed method performs well as shown in Fig.3.

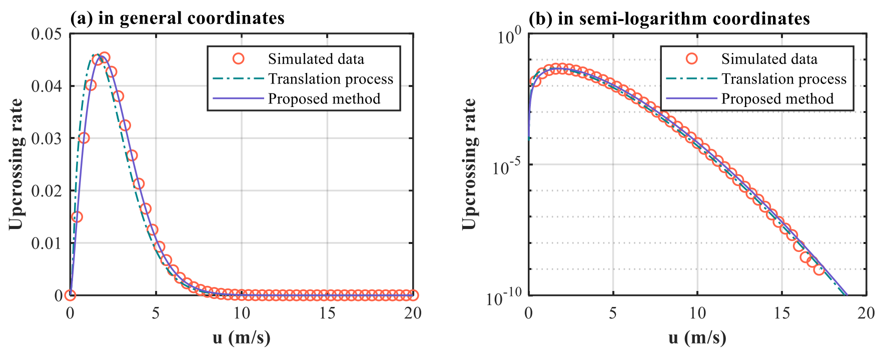

Figure 3. The comparison of the upcrossing rates of the simulated mean wind speeds following the Weibull distribution, translation process method and proposed method

4. CONCLUSIONS

This study proposes a novel method for estimating the upcrossing rate of the mean wind speed process following the Weibull distribution, and this method is also applicable for a discrete random process with other distributions. There is a difference between this method's estimates and the analytical expression of the upcrossing rate of the discrete standard Rayleigh process. The simulated data suggests the proposed method has a high accuracy for estimating the upcrossing rate.

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REFERENCES

Harris, R. I., 2014. A simulation method for the macro-meteorological wind speed and the implications for extreme value analysis. Journal of Wind Engineering and Industrial Aerodynamics, 125, 146-155.

Harris, R. I. and Cook, N. J., 2014. The parent wind speed distribution: Why Weibull?. Journal of Wind Engineering and Industrial Aerodynamics, 131, 72-87.

Harris, R. I., 2017. The Level Crossing Method applied to mean wind speeds from “mixed” climates. Structural Safety, 67, 54-61.

Liu, P. L. and Der Kiureghian, A., 1986. Multivariate distribution models with prescribed marginals and covariances. Probabilistic Engineering Mechanics, 1(2), 105-112.

Liu, M., Peng, L., Huang, G., Yang, Q. and Jiang, Y., 2020. Simulation of stationary non-Gaussian multivariate wind pressures using moment-based piecewise Hermite polynomial model. Journal of Wind Engineering and Industrial Aerodynamics, 196, 104041.

Nelsen, R.B., 2006. An introduction to copulas, Springer Series in Statistics seconded. Springer-Verlag, New York.

Shinozuka, M. and Deodatis, G., 1991. Simulation of stochastic processes by spectral representation.

Van der Hoven, I., 1957. Power spectrum of horizontal wind speed in the frequency range from 0.0007 to 900 cycles per hour. Journal of Atmospheric Sciences, 14(2), 160-164.